B.Sc - III ① Topic Dr. Lalan Kumar Singh;
Paper - VII . ATTRACTION of ROD Deptt. of Mathematics
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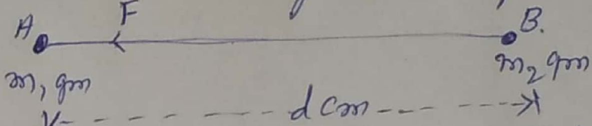
Newtonian Law of Attraction

"Every particle of matter attracts every other particle of matter with a force, which varies directly as the product of the masses of the particles and inversely as the square of the distance between them."



Let m_1 gm. and m_2 gm. be the masses of the two particles A & B. and distance between A & B = d cm.

Then according to Newton's Law of Attraction, if F be the attraction between matters A & B defined as

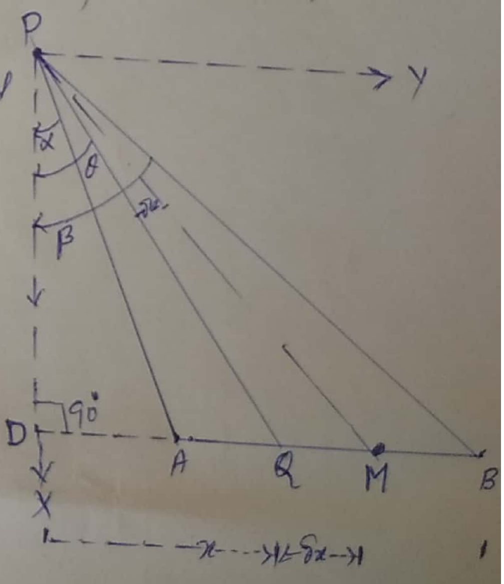
$F \propto m_1 \cdot m_2$ & $F \propto \frac{1}{d^2}$ where, m_1, m_2 & d are constants

$\therefore$ By theorem of joint variation, $F \propto \frac{m_1 \cdot m_2}{d^2}$

i.e $F = \gamma \frac{m_1 \cdot m_2}{d^2}$ dynes

Where the quantity γ is called the constant of attraction.

Theorem:- Find the attraction of a thin uniform rod upon an external point.



Solution:- Let AB be the thin uniform rod and P the external point at, at which the attraction of AB to be found out. Draw PD from point P, perpendicular to AB, let PD(= p).

Now we take any point Q on AB, such that DQ = x and $\angle DPQ = \theta$.

Let QM be an element δx of the rod, k the cross-section and ρ the density of the rod.

Then the mass of QM = $k\rho\delta x$.

Now, from ΔPDQ, $\sec\theta = \frac{PQ}{p}$, or $PQ = p\sec\theta$.

and $\tan\theta = \frac{x}{PQ} = \frac{x}{p}$ or $x = p\tan\theta$. $\therefore \delta x = p\sec^2\theta \cdot \delta\theta$.

Now the attraction of QM on a unit of mass at point P =

$= \frac{\gamma \cdot 1 \cdot k\rho\delta x}{PQ^2} = \frac{\gamma k\rho \cdot p\sec^2\theta\, \delta\theta}{p^2\sec^2\theta} = \frac{\gamma k\rho\,\delta\theta}{p}$.

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→ Since QM is very small and the rod is very thin, therefore this attraction $\frac{\gamma k \rho \delta\theta}{p}$ acts ultimately along PQ.

Now its components will be along PD and perpendicular to PD. $\frac{\gamma k\rho\delta\theta}{p}\cos\theta$ and $\frac{\gamma k\rho\delta\theta}{p}\sin\theta$ respectively.

Again, Let $\angle DPA = \alpha$ and $\angle DPB = \beta$ and X, Y be the component attraction of the whole rod AB, (∥ PD and ⊥ PD) along PD and perpendicular to PD respectively.

Then $X = \int_{\alpha}^{\beta} \frac{\gamma k\rho}{p}\cos\theta\, d\theta = \frac{\gamma k\rho}{p}\left[\sin\theta\right]_{\alpha}^{\beta} = \frac{\gamma k\rho}{p}(\sin\beta - \sin\alpha)$.

and $Y = \int_{\alpha}^{\beta} \frac{\gamma k\rho}{p}\sin\theta\, d\theta = \frac{\gamma k\rho}{p}\left[-\cos\theta\right]_{\alpha}^{\beta} = \frac{\gamma k\rho}{p}(\cos\alpha - \cos\beta)$.

Let R be the resultant attraction inclined at an angle γ to PD.

then $R = \sqrt{X^2+Y^2} = \frac{\gamma k\rho}{p}\sqrt{(\sin\beta-\sin\alpha)^2+(\cos\alpha-\cos\beta)^2}$

$= \frac{\gamma k\rho}{p}\sqrt{(\sin^2\beta+\cos^2\beta)+(\sin^2\alpha+\cos^2\alpha)-2(\cos\alpha\cdot\cos\beta+\sin\alpha\cdot\sin\beta)}$

$= \frac{\gamma k\rho}{p}\sqrt{2-2\cos(\beta-\alpha)} = \frac{\gamma k\rho}{p}\sqrt{2(1-\cos(\beta-\alpha))}$

$= \frac{\gamma k\rho}{p}\sqrt{2\cdot 2\cdot\sin^2\frac{\beta-\alpha}{2}} = \frac{2\gamma k\rho}{p}\sin\frac{\beta-\alpha}{2}$

$\therefore R = \frac{2\gamma k\rho}{p}\sin\left(\frac{APB}{2}\right)$ ——— (I)

And $\tan\gamma = \frac{Y}{X} = \frac{\cos\alpha-\cos\beta}{\sin\beta-\sin\alpha}$

$= \frac{2\sin\frac{\alpha+\beta}{2}\cdot\sin\frac{\beta-\alpha}{2}}{2\cos\frac{\alpha+\beta}{2}\cdot\sin\frac{\beta-\alpha}{2}} = \tan\frac{\alpha+\beta}{2}$

$\therefore \tan\gamma = \tan\frac{\alpha+\beta}{2}$

$\Rightarrow \gamma = \frac{\alpha+\beta}{2}$ ——— (II)

Eqn (II) ⇒ The resultant attraction bisects the angle APB.

Corollary :- Find the attraction of an infinite rod upon an external point.

Solution :- Since the rod AB is infinite in length

$\therefore \alpha = -90^\circ$ and $\beta = 90^\circ$

$\therefore R = \frac{2\gamma k\rho}{p}\sin\frac{\beta-\alpha}{2} = \frac{2\gamma k\rho}{p}\sin 90^\circ = \frac{2\gamma k\rho}{p}\cdot 1$.

Hence the attraction of an infinite rod on an external point varies inversely as the distance of the point from the rod.

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