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Definition of Group:

A nonempty set G with a binary operation 'o' is called a group (G, o) if the following properties holds for the operation 'o'.

① Closure property:- G is a closed under the binary operation 'o'.

i.e $a \circ b \in G \ \forall \ a, b \in G$.

② Associative property:— The associative property holds for the operation 'o'

i.e, $(a \circ b) \circ c = a \circ (b \circ c) \ \forall \ a, b, c \in G$.

③ Existence of Identity Element

The identity element exists in G, i.e, $a \circ e = e \circ a = a \ \forall \ a \in G$.

④ Existence of Inverse Element:-

for every $a \in G$, $\exists$ an element $b \in G$ such that $a \circ b = b \circ a = e$, where e being the identity element.

The (G, o) is a group.

The inverse element of a is denoted by a^{-1} and so

$a \circ a^{-1} = a^{-1} \circ a = e$.

Theorem:- Prove That identity element is a group is unique.

PROOF:- Let e be the identity element in group (G, o).

Then existence of Identity element of the group, we have

$$a \circ e = e \circ a = a \quad \text{—— (i)}$$
$$\forall a \in G$$

Now to prove that e is unique

Let e is not unique. Let e' be another identity element in a group (G, o) where $e' \neq e$.

Then existence of Identity element of the group, We have.

$$a \circ e' = e' \circ a = a \quad \text{—— (ii)}$$
$$\forall a \in G.$$

Since (i) is true $\forall a \in G$ and $e' \in G$

therefore putting $a = e'$ in (i)

$$e' \circ e = e \circ e' = e' \quad \text{—— (iii)}$$

Similarly, putting $a = e$ in (ii)

we get $e \circ e' = e' \circ e = e$ —— (IV)

From eqn (iii) and (IV)

$e' = e$, which is contradicts our assumption that $e' \neq e$. Hence identity element is unique

Dosage and administration

A Given two properties ~~are~~ are satisfied then Group (G, o) is Called Semi-Group.

A 180 Commutative property hold in Group (G, o)

i.e $a, b \in G$, then

$a o b = b o a$, $\forall\ a, b \in G$

Then the Group (G, o) is called Abelian Group.

for Example-

(i) $\{1\ \omega\ \omega^2\}$ is a finite Group w.r.to multiplication. Where ω is the Cube roots of unity.

(ii) The set of all integers is an finite group w.r. to addition.