8/04/20

MATRIX

(A) Examples: $[2]$, $\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$ $\begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 4 \end{bmatrix}$.

These are the examples of Matrices

(B) Row Matrix $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}_{1\times 3}$

(C) Column Matrix $\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}_{3\times 1}$

(D) Rectangular Matrix $\begin{bmatrix} 2 & 3 & 4 \\ -1 & 0 & 2 \end{bmatrix}_{2\times 3}$

(E) Square Matrix $\begin{bmatrix} -1 & 0 & 1 \\ 2 & 3 & -2 \\ -2 & 3 & 5 \end{bmatrix}_{3\times 3}$

(F) Diagonal Matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 4 \end{bmatrix}_{3\times 3}$

Non-diagonal elements zero

(G) Scalar Matrix $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$,

$\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}_{2\times 2}$

(H) Identity Matrix : $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$

(I) Zero Matrix = $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times 2}$, $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3\times 3}$

J)

(4) 18/04/20.

Invertible Matrices

If A is a square matrix of order m, and there exist another square Matrix B of the same order such that

$$AB = BA = I$$

then B is called inverse of A.

i.e $A^{-1} = B$

example $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

Find AB & BA

$$\boxed{(AB)^{-1} = B^{-1}A^{-1}}$$

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Operations on Matrices (i) 18/04/2020

(a) If A, B, C are three Matrices of same order then

(i) $A + B = B + A$

(ii) $A + (B + C) = (A + B) + C$

(iii) $A + O = O + A = A$

(iv) $A + (-A) = O$ (zero Matrix)

(b) (i) $K(A + B) = KA + KB$

(ii) $(AB)C = A(BC)$

(iii) $A(B + C) = AB + AC$

(iv) $(A + B)C = AC + BC$

(v) $IA = AI = I$

(c) Transpose of Matrix

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 0 & -1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 3 & 0 \\ 2 & 4 & -1 \end{bmatrix}$$

properties of transpose of the Matrices (iii) 18/04/2020

(i) $(A')' = A$

(ii) $(KA)' = KA'$

(iii) $(A+B)' = A' + B'$

(iv) $(AB)' = B'A'$

(D) (i) Symmetric Matrix If

if $A = A'$

i.e $A - A' = 0$ (zero Mat

(ii) Skew symmetric If

$A = -A'$ i.e $A' = -A$

$\Rightarrow A' + A = 0$

(E) Every square Matrix can be expressed as the ~~same~~ sum of symmetric and a skew symm - Matrix. For Matrix A

$A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A') = P + Q$

When P is symm & Q skew symm

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Row Matrix $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}_{1\times 3}$

Column Matrix $\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}_{3\times 1}$

Diagonal Matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}_{3\times 3}$

Scalar Matrix $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & & 2 \end{bmatrix}_{3\times 3}$

Identity Matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}$

Square Matrix $\begin{bmatrix} & \end{bmatrix}_{m\times m}$

Simplify / Evaluate 10/04/2020.

Q. $\sin^{-1}(x^2\sqrt{1-x^2} + x\sqrt{1-x^4})$

$$\boxed{\sin^{-1}x+\sin^{-1}y = \sin^{-1}(x\sqrt{1-y^2}+y\sqrt{1-x^2})}$$

Sol. Let $x = \sin\beta$ $\quad x^2 = \sin^2\beta \Rightarrow 1-x^2$

$\beta = \sin^{-1}x$

$\cancel{x^2 = \sin\alpha}$ $\Rightarrow \sqrt{1-x^2} = \cos\beta$

Similarly $x^2 = \sin\alpha \Rightarrow x^4 = \sin^2\alpha$

$\alpha = \sin^{-1}x^2 \Rightarrow 1-x^4 = 1-\sin^2\alpha$

$\sqrt{1-x^4} = \cos\alpha.$

$\sin^{-1}(\sin\alpha\sqrt{1-\sin^2\beta} + \sin\beta\sqrt{1-\cos^2\alpha})$

$\Rightarrow \sin^{-1}\{\sin\alpha\cdot\cos\beta+\cos\alpha\cdot\sin\beta\}$

$\Rightarrow \sin^{-1}\{\sin(\alpha+\beta)\}$

$= \cancel{\sin}\ \alpha+\beta$

$= \sin^{-1}x^2+\sin^{-1}x$

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