Date- 19/04/2020 **Complex Numbers** Dr. Lalan Kumar Singh Maths.

(B.Sc – I & II) Hons/Sub.

Here we deal Complex numbers with "iota" i.e $i = \sqrt{-1}$

Firstly, why do I need a Complex numbers?

We take a quadratic equation $x^2+1=0$ obviously here $a=1, b=0 \,\&\, c=1$. When we start to solve we get $x^2=-1$. → $\Rightarrow x = \pm\sqrt{-1}$. But $\sqrt{-1}$ does not exist in Real number i.e R.

$\sqrt{-1}$ is denoted by i called "iota".

Numbers of the form $a+ib$ are called Complex numbers, where $a \,\&\, b$ are real numbers and $i=\sqrt{-1}$. In we write $(x+iy)$ x – real part & y – imaginary part.

Conjugate :- The Complex numbers $x-iy$ $\underbrace{\{x+(-i)y\}}$ and $(x+iy)$ are said to be (defined) as conjugate to each other.

Some properties of complex numbers:-

i ~~this numbers~~ If $x+iy=0 \Rightarrow x=0$ and $y=0$. $\{0+i0$ is a complex no.$\}$

ii If $x_1+iy_1 = x_2+iy_2 \Rightarrow x_1 = x_2$ and $y_1=y_2$

iii If $x_1+iy_1 = x_2+iy_2 \Rightarrow x_1-iy_1 = x_2-iy_2$, and conversely.

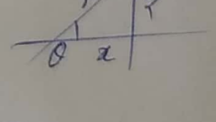
Modulus and Argument

We put $x+iy = r(\cos\theta + i\sin\theta)$, Here r and θ are real and to be determined. Equating real & imaginary part, we get

$x = r\cos\theta$, $y = r\sin\theta$ $\therefore x^2+y^2 = r^2(\cos^2\theta+\sin^2\theta) = r^2$

Which implies $r = \sqrt{x^2+y^2}$ ——— (I) Called Modulus.

Again $y/x = \dfrac{r\sin\theta}{r\cos\theta} = \tan\theta$, ——— (II)



we have

$\cos\theta = \dfrac{x}{r}$ & $\sin\theta = y/r$

$\therefore \cos\theta = \dfrac{x}{r} = \dfrac{x}{\sqrt{x^2+y^2}}$ and $\sin\theta = y/r = \dfrac{y}{\sqrt{x^2+y^2}}$

(Here the positive value of r to be taken & value of θ lies between $(\pi \,\&\, -\pi)$.

In general form $r\{\cos(2n\pi+\theta) + i\sin(2n\pi+\theta)\} = x+iy$.

We see $(\cos\theta + i\sin\theta)$ remains unchanged if we put $\theta = \theta + 2n\pi$, where $n \in I$ i.e n is an integer.

→ Above equation $x^2+1=0 \Rightarrow x^2=-1$, we know that in Real number system square of any number is not –ve.

Theorem:- State and Prove De Moivre's theorem →.

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Statement of De Moivre's theorem:-

(i) If n be the positive integer or a negative integer, then
$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$. $\{n \text{ is } + \text{ or } -\}$

(ii) If n be a fraction, positive or negative, then one of the value of $(\cos\theta + i\sin\theta)^n$ is $\cos n\theta + i\sin n\theta$. $\{n \text{ is of the form } p/q,\ q \neq 0.\}$

Proof:- Case-I. Let n be a positive Integer. $(n>0)$

We have, by actual multiplication. $(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)$

$= (\cos\theta_1\cos\theta_2 - \sin\theta_1 \cdot \sin\theta_2) + i(\sin\theta_1\cos\theta_2 + \cos\theta_1 \cdot \sin\theta_2)$

$= \cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)$ In this way to proceed we get

Also, $(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)(\cos\theta_3 + i\sin\theta_3)$

$= \{\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\}(\cos\theta_3 + i\sin\theta_3)$

$= \cos(\theta_1+\theta_2+\theta_3) + i\sin(\theta_1+\theta_2+\theta_3)$.

The product of n similar factors is

$(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)(\cos\theta_3 + i\sin\theta_3) \cdots\cdots$

$\cdots (\cos\theta_{n-1} + i\sin\theta_{n-1})(\cos\theta_n + i\sin\theta_n)$.

$= \cos(\theta_1+\theta_2+\theta_3+\theta_4+\cdots+\theta_n) + i\sin(\theta_1+\theta_2+\theta_3+\cdots\theta_n)$

Now we put $\theta_1 = \theta_2 = \theta_3 = \cdots = \theta_n = \theta$ (each), we get

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$. —— (A)

Hence the proof this theorem, when n is a positive integer.

Case-2 when n is negative. $n<0$

Case-3. when n is of the form p/q, $q \neq 0$ i.e a fraction

→ next day. Thanks

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