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B.Sc. Part I (Hons)

Date - 10.06.2021

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Time :- 11.30-12.15

Topic :— Partial Differentiation

Let $u = f(x, y)$, i.e let u be a function of two variable x and y. Then $\lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}$ is exists is called the partial derivative or partial diff. co-efficient of $f(x, y)$ w.r.to x and is denoted by the

$\frac{\delta f}{\delta x}$ or $\frac{\partial u}{\delta x}$ or f_x, u_x.

V.V.Imp. Theorem

Euler's Theorem

Statement :— If $f(x, y)$ be a homogeneous function of degree n, Then

$$x\frac{\delta f}{\delta x} + y\frac{\partial f}{\partial y} = nf.$$

Proof :- Let $f(x,y) = Ax^{\alpha_1}y^{\beta_1} + Bx^{\alpha_2}y^{\beta_2} + Cx^{\alpha_3}y^{\beta_3} + \cdots$ (1)

Where $\alpha_1+\beta_1 = \alpha_2+\beta_2 = \alpha_3+\beta_3 = \cdots = n$ (ii)

and $A, B, C, \ldots$ are constants.

Diff. eqn (1) partially w.r.to x we get

$$\frac{\partial f}{\partial x} = A\alpha_1 x^{\alpha_1-1} y^{\beta_1} + B\alpha_2 x^{\alpha_2-1} y^{\beta_2} + C\alpha_3 x^{\alpha_3-1} y^{\beta_3} + \cdots$$

$$\therefore x\frac{\partial f}{\partial x} = A\alpha_1 x^{\alpha} y^{\beta_1} + B\alpha_2 x^{\alpha_2} y^{\beta_2} + C\alpha_3 x^{\alpha_3} y^{\beta_3} + \cdots \quad \text{(III)}$$

Again diff. (1) partially w.r.to y, we get

$$\frac{\partial f}{\partial y} = A\beta_1 x^{\alpha_1} y^{\beta_1-1} + B\beta_2 x^{\alpha_2} y^{\beta_2} + C\beta_2 x^{\alpha_2} y^{\beta_2} + \cdots \quad \text{(IV)}$$

Adding eqn (III) and (IV)

We get

$$x\frac{\partial f}{\partial y} + y\frac{\partial f}{\partial y} = Ax^{\alpha_1}y^{\beta_1}(\alpha_1+\beta_1) + Bx^{\alpha_2}y^{\beta_2}(\alpha_2+\beta_2) + Cx^{\alpha_3}y^{\beta_3}(\alpha_3+\beta_3) + \cdots$$

$$= Ax^{\alpha_1}y^{\beta_1}\cdot n + Bx^{\alpha_2}y^{\beta_2}\cdot n + Cx^{\alpha_3}y^{\beta_3}\cdot n + \text{---}$$

by eqn (1)

$$= n\left(Ax^{\alpha_1}y^{\beta_1} + Bx^{\alpha_2}y^{\beta_2} + Cx^{\alpha_3}y^{\beta_3} + \cdots\right)$$

$$= nf(x,y)$$

Hence By (1)

$$x\frac{\partial f}{\partial y} + x\frac{\partial f}{\partial y} = nf(x,y)$$

Proved.