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$$E_{n_1} = - \frac{2\pi^2 m e^4}{n_1^2 h^2}$$

$$E_{n_2} = - \frac{2\pi^2 m e^4}{n_2^2 h^2}$$

The difference of energy between the levels n_1 and n_2 is

$$\Delta E = E_{n_2} - E_{n_1} = \frac{2\pi^2 m e^4}{h^2} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \quad (i)$$

According to Planck's constant

$$\Delta E = h\nu = \frac{hc}{\lambda} \quad (ii)$$

Where λ is wavelength of photon and c is velocity of light. From equation (i) and (ii) we can write

$$\frac{hc}{\lambda} = \frac{2\pi^2 m e^4}{h^2} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$\text{or } \frac{1}{\lambda} = \frac{2\pi^2 m e^4}{c h^3} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$\text{or } \frac{1}{\lambda} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

Where R is Rydberg constant.

The value of R can be calculated⁽⁵⁾ as the value of $e, m, h,$ and c are known. It comes out to be $109,679 \text{ cm}^{-1}$ and agrees closely with the value of Rydberg constant in the original empirical Balmer's equation $109,677 \text{ cm}^{-1}$.

Calculation of wave lengths of the spectral lines of Hydrogen in the visible region. These lines constitute the Balmer series when $n_1 = 2$, now the equation

$$\frac{1}{\lambda} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \text{ can be written as}$$
$$\frac{1}{\lambda} = 109679 \left(\frac{1}{2^2} - \frac{1}{n_2^2} \right)$$

Thus the wave lengths of the photons emitted as the electron returns from energy levels 7, 6, 5, 4 and 3 were calculated by Bohr. The calculated values corresponding exactly to the values of wave lengths of the spectral lines already known. This was in fact, a great success of the Bohr atom.

Example - Find the wave length in \AA of the line in Balmer series that is associated with drop of the electron from the 4th orbit. The value of Rydberg constant is 109676 cm^{-1} .

Solution: - The wave lengths of lines in Balmer series are given by

$$\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n^2} \right)$$

where $R = 109676 \text{ cm}^{-1}$ $n = 4$

$$\begin{aligned} \frac{1}{\lambda} &= 109676 \left(\frac{1}{2^2} - \frac{1}{4^2} \right) \\ &= 109676 \left(\frac{16-4}{64} \right) \end{aligned}$$

$$= 109676 \times \frac{12}{64} = 109676 \times \frac{3}{16}$$

$$\lambda = \frac{16}{109676 \times 3}$$

$$= 4862 \times 10^{-8} \text{ cm}$$

$$= 4862 \times 10^{-8} \times 10^8 \text{ \AA}$$

$$= 4862 \text{ \AA}$$