Dr. Lalan Kumar Singh, Maths

B.Sc - I (Hons) (Sub) Dt 26·4·21 - 11·30 – 12·15
Topic – The Plane (Three dimensional Geometry)

Plane is nothing but a locus of a line with Conditions. If a St line (lying (x, y) Co-ordinate in XY plane) sweeping along z axis then the locus of the line Makes a Surface Called plane.

Theorem: Prove that every linear equation in Three variables x, y, z always represents a plane.

Proof:- We assume a linear equation in Three ~~degree~~ variables x, y and z

i.e. $ax + by + cz + d = 0$ ——— (1)

Here a, b, c & d are Constants and one of the Constants a, b and c is not zero.

Now, Let (x_1, y_1, z_1) and (x_2, y_2, z_2) be the Co-ordinates of any two points lying on the locus of eqn (1)

then $ax_1 + by_1 + cz_1 + d = 0$ ——— (2)

& $ax_2 + by_2 + cz_2 + d = 0$ ——— (3)

We multiply eqn (2) by m_2 and eqn (3) by m_1 and adding

We get $(ax_1 + by_1 + cz_1 + d)m_2 + (ax_2 + by_2 + cz_2 + d)m_1 = 0$

$\Rightarrow a(m_1x_1 + m_2x_2) + b(m_1y_2 + m_2y_1) + c(m_1z_2 + m_2z_1) + d(m_1 + m_2) = 0$

$\Rightarrow a(m_1x_1 + m_2x_2) + b(m_1y_2 + m_2y_1) + c(m_1z_2 + m_2z_1) + d(m_1 + m_2)$

$\Rightarrow a\dfrac{(m_1x_1 + m_2x_2)}{m_1 + m_2} + b\dfrac{(m_1y_2 + m_2y_1)}{m_1 + m_2} + c\cdot\dfrac{(m_1z_2 + m_2z_1)}{m_1 + m_2} + d =$

It means:- the points $\left(\dfrac{m_1x_1 + m_2x_2}{m_1 + m_2}, \dfrac{m_1y_2 + m_2y_1}{m_1 + m_2}, \dfrac{m_1z_2 + m_2z_1}{m_1 + m_2}\right)$ when $m_1 + m_2 \neq 0$ satisfy the equation (1) Mean, this point lie on the locus of eqn (1)

Hence the line joining (x_1, y_1, z_1) and (x_2, y_2, z_2) Completely lies on the locus of (1) This ~~points~~ ~~def~~ divides the line joining in the ratio $m_1 : m_2$ also.

Hence this locus is a plane.

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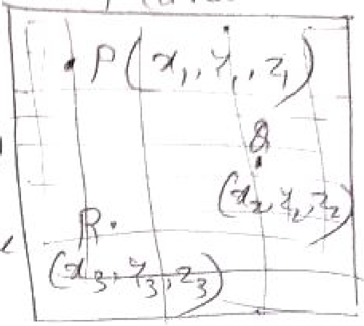
Topic. The plane in 3D.

Theorem:- Find the equation of a plane passing through three given non-collinear points.

Answer:- We assume three given non collinear points are P, Q and R whose co-ordinates are (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) respectively

Plane



Let the equation of the plane through these points be $ax + by + cz + d = 0$ ——— (1)

Now, since the point (assumed) lie in the this plane eqn (1) the co-ordinates of the points P, Q & R must satisfy the equn (1). i.e,

$$ax_1 + by_1 + cz_1 + d = 0 \quad \text{——— (2)}$$

$$ax_2 + by_2 + cz_2 + d = 0 \quad \text{——— (3)}$$

$$ax_3 + by_3 + cz_3 + d = 0 \quad \text{——— (4)}$$

We have to eliminate constants, a, b, c & d. by which.

$$\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = 0 \quad \text{——— (5)}$$

Other method, eqn (2)-(3), (3-4) (4-2) which is lazy process.

Equation (5) is of the first degree in x, y, z and therefore represents a plane. Hence the result.

$R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$, $R_3 \to R_3 - R_4$ we get

$x - x_1$